Equitability Through Repeated Committee Selection: Algorithms, Complexity, and Visualization

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Abstract. We study the problem of finding a sequence of committees for a given election with approval preferences that balances the agents' satisfactions as much as possible. For minimization, we consider both the maximum difference and the sum of differences over the satisfactions. We settle the problems' computational complexity against various restrictions like the size of the approval sets. In this regard, we also show sharp tractability borders with NP-hardness on the one side and polynomial-time algorithms on the other. We complement our theoretical analysis with an empirical study. Therein, we give a two-dimensional visualization of elections, with various advantages over the established *maps of elections* framework.

1 Introduction

The problem of making collective selections in a fair and satisfying manner arises in a wide range of contexts, from civic committees and collaborative teams to academic planning. Whenever multiple stakeholders wish to see their interests represented in a chosen subset, ensuring perceived fairness and balanced satisfaction becomes essential. As a concrete illustration, consider academic settings where departments, working groups, or external partners collectively select panels, highlight topics, or coordinate initiatives—but the underlying challenges and solutions discussed here are relevant in many similar environments.

A persistent challenge in these settings arises when a single, static selection inevitably leaves some participants unsatisfied, because it is not possible to represent all interests fairly at once. To address this, we propose a flexible approach where the original selection may be "unfolded" across several occasions, but *only* to the extent *considered necessary or desirable* for achieving a fairer representation. The key innovation is that the number of rounds or committees is not fixed in advance; instead, the sequence is introduced as a tool and its length can be adapted to facilitate fairer or more balanced outcomes.

Illustrative Example. Suppose we have $n = 9$ departments and $m = 8$ proposed topics for outreach activities during the next year. Each department submits their approval ballots as follows:

$1: \{a, c\}$	$2: \{b, d\}$	$3: \{a, b\}$
$4: \{c, d\}$	$5: \{a, d\}$	$6: \{b, c\}$
$7: \{e, f\}$	$8: \{e, g\}$	$9: \{e, h\}$

Suppose we wish to select three topics (committee of size $k = 3$). No single size-3 committee can satisfy each voter: Some will see none of their approved candidates chosen. However, by allowing a repeated selection of three (possibly different) topics for each of the next two years (i.e., a sequence of two committees, like $(\{a, b, e\}, \{c, d, e\})$), one can guarantee that *every* voter attains the same total satisfaction of exactly 2, either by having one (possibly the same) approved topic chosen in each year, or two approved topics selected in a single year.[1] Similarly, the original plan could be to choose three topics for the next two (or more) years once, while our repeated committee selection approach could consider a more fine-grained time frame by splitting the overall time frame into multiple, shorter time windows (e.g., year 1 and 2) and selecting three topics for each time window. Both the described expanding and splitting of a time frame can be modelled as repeated committee selections and they are technically the same in our model.

The central question we address is: *Given a set of approval ballots, how can we construct a sequence of committees such that the overall satisfaction is as equally distributed as possible among all participants?* We analyze the computational complexity of this problem, develop algorithms, and empirically explore typical instances.

1.1 Preliminaries and Problem Definitions

Let $\mathbb{N}$ and $\mathbb{N}_0$ be the natural numbers excluding and including zero, respectively, $[t]$ the set $\{1, \dots, t\}$ for any integer t, and $\mathcal{P}(X)$ the power set of any set X.

We consider elections with approval preferences, i.e., elections of the form $\mathcal{E} = (A, C, U, k)$ consisting of a set A of n agents, a set C of m candidates, an approval profile $U\colon A \to \mathcal{P}(C)$, and a desired committee size $0 < k \leq m$. As we are interested in planning in advance, i.e. for multiple *repetitions* of an election, we define

$$\mathcal{R}(\mathcal{E}, r) \coloneqq \{(X_1, \dots, X_r) \mid \forall t \in [r] : |X_t| = k \wedge X_t \subseteq C\}$$

as all possible *committee sequences* for r repetitions.

Aiming for a number of repetitions so that the *satisfactions* of the agents are *equitable* to a certain degree, we need to define the considered satisfaction and equitability measures first. We investigate two satisfaction models: $\mathrm{sat}_{\mathrm{base}}$ measures the satisfaction of an

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[1] Interestingly, while there is an equitable committee sequence for two repetitions with $k = 3$ for each, there is none for $k = 6$ and one repetition (i.e. one committee selection) in our example.

agent $a \in A$ via the number of approved candidates that are part of $\mathcal{X} \in \mathcal{R}(\mathcal{E}, r)$, while $\mathrm{sat}_{\mathrm{norm}}$ is a normalized version of $\mathrm{sat}_{\mathrm{base}}$ where the satisfaction is weighted by the overall number of candidates approved by a:

$$\mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a) \coloneqq \sum_{t \in \{1,\dots,r\}} |U(a) \cap X_t|$$

$$\mathrm{sat}_{\mathrm{norm}}(\mathcal{E}, \mathcal{X}, a) \coloneqq \frac{\mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a)}{|U(a)|}.$$

Thus, we assume that both the candidate set and the approval profiles do not change when repeating the election.

To measure the equitability of the satisfactions for a committee sequence $\mathcal{X} \in \mathcal{R}(\mathcal{E}, r)$, we consider two measures as well: For a given satisfaction measure $\mathrm{sat}_{\mathrm{s}}$, MD_{s} is the maximum difference in the satisfactions and SD_{s} is sum of the differences of the satisfactions, i.e.:

$$\mathrm{MD}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}) = \max_{a,a' \in A} |\mathrm{sat}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}, a) - \mathrm{sat}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}, a')|$$

$$\mathrm{SD}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}) = \sum_{\{a,a'\} \subseteq A} |\mathrm{sat}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}, a) - \mathrm{sat}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}, a')|.$$

Therefore, the smaller the value, the higher the equitability of the satisfactions. In addition, the smallest value is zero, reached only when all agents have the same satisfaction. These measures have also been used as equitability measures in the context of the facility location problem—see [25] for an overview of equitability measures in that context.

Next, we formally define our central problem of finding a number of repetitions so that the equitability value is at most some given δ:

Problem 1 (γ-EXISTENTIAL EQUITABILITY, short γ-EEQI)**.** Given an election $\mathcal{E}$ and a bound $\delta \in \mathbb{N}_0$ on the equitability measured with γ, is there an $r \geq 1$ and a committee sequence $\mathcal{X} \in \mathcal{R}(\mathcal{E}, r)$ such that $\gamma(\mathcal{E}, \mathcal{X}) \leq \delta$?

In the following, we omit arguments if they are clear from context.

1.2 Our Contributions

We study the computational complexity of γ-EEQI against several parameter restrictions (see Table 1 for an overview). We outline some highlights. We show that for $\mathrm{sat}_{\mathrm{norm}}$, both problems are polynomial-time solvable. In contrast, for $\mathrm{sat}_{\mathrm{base}}$, we detect several intractable cases and show that $\mathrm{MD}_{\mathrm{base}}$ and $\mathrm{SD}_{\mathrm{base}}$ (presumably) differ, for instance regarding the parameter k, where for $k = 1$ $\mathrm{MD}_{\mathrm{base}}$ is polynomial-time solvable and $\mathrm{SD}_{\mathrm{base}}$ is **NP**-hard. For $k = 2$, $\mathrm{MD}_{\mathrm{base}}$ becomes **NP**-hard as well. We prove sharp tractability borders regarding the sizes and the differences of the sizes of the approval sets. Herein, both problems are polynomial-time solvable if the approval sets are of size one, yet become **NP**-hard if the largest approval set is of size two. Similarly for the difference: both problems are polynomial-time solvable when every agent approves the same number of candidates, but becomes **NP**-hard when the maximum difference in the sizes of any two approval sets is one. While both $\mathrm{MD}_{\mathrm{base}}$ and $\mathrm{SD}_{\mathrm{base}}$ are **NP**-hard even for constant values of the aforementioned parameters, we show that they become uniformly polynomial-time solvable for constant values of the number n of agents or of the number m of candidates.

We also present numerical experiments which investigate the *most equitable* committee sequence and the minimal number of repetitions required to achieve it. For $\mathrm{MD}_{\mathrm{base}}$ and $\mathrm{SD}_{\mathrm{base}}$, the best equitability value can differ significantly between different elections. Often (but not always), only one repetition (i.e. one voting round) is needed when measuring the equitability with $\mathrm{MD}_{\mathrm{base}}$. In contrast, there are significantly more elections requiring more than one repetition when using $\mathrm{SD}_{\mathrm{base}}$. We observe that some of these elections share the following properties with some real-world approval elections collected in Pabulib [12]: 1) The sum of approval score of the candidates $w(\mathcal{E})$ relative to the highest possible value for the given election dimension and 2) the maximal difference of the approval scores $e(\mathcal{E})$ relative to its highest possible value. In addition, our experimental results regarding equitability differ for different $w(\mathcal{E})$ and $e(\mathcal{E})$ values.

We therefore propose visualizing elections in two-dimensional space by determining their coordinates through w and e. Our (mainly empirical) comparison with the *Map of Approval Elections* [30]—which is state-of-the-art for visualizing approval elections—shows several benefits of our approach.

Several details (e.g. proofs marked with $\star$) are deferred to the full version of the paper.

2 Related Work

Committee selection lies at the heart of computational social choice, motivated by a variety of goals such as individual excellence, proportionality, and diversity [8]. Our approach is closely related to the pursuit of diversity and egalitarianism, where the objective is to maximize the minimum satisfaction among agents [1].

Committee Sequences. We extend the classical view by considering the selection of a sequence of committees. Related so-called *multistage* problems have explored cases where agents possess multiple preferences or where there are both satisfaction constraints in each round and similarity constraints between consecutive committees [18, 4, 7, 14, 6, 32, 9]. Distinct from these mostly dynamic settings, our repetitive setting focuses on aggregate satisfaction across the full sequence, with constant preferences. Models for sequential group decisions have also been examined by Bredereck et al. [3], who introduce a time dimension but with static preferences, and by Bulteau et al. [5], who target justified group representation in offline settings. Kahana and Hazon [17] show that the leximin approach provides an approximate form of proportionality, if such outcomes exist. Online variants with evolving ballots [13, 20, 28] differ substantially from our offline scenario and generally focus on single-winner decisions or employ different solution quality assessments. For an overview on the related topic of temporal multiwinner elections, we refer to a survey by Elkind et al. [10].

Repeated Collective Decisions. Scenarios in which a collective decision unfolds over multiple, possibly sequential rounds, are increasingly prominent. This is especially apparent in participatory budgeting [21] or fair allocation [16], where the allocation of resources or selection of projects is repeated over time. Indeed, our setting is closest to that of Igarashi et al. [16], who consider a sequence of allocations of indivisible goods to agents in order to minimize envy between the agents. In a flavor similar to our repetition of committees or the repetition of allocations, Igarashi et al. [15] have analyzed the allocation of agents to activities with some activities being "copyable".

Equitability Measures. A central objective in the design of fair committee selection rules—both in classical and repeated settings—is to aggregate individual preferences in an equitable manner. Various measures have been developed for evaluating the distribution of satisfaction among agents. The most prominent are minimum

Table 1. Overview of our theoretical results.

Parameter	γ-EEQI		
	$\gamma \in \{\mathrm{MD}_{\mathrm{norm}}, \mathrm{SD}_{\mathrm{norm}}\}$	$\gamma = \mathrm{MD}_{\mathrm{base}}$	$\gamma = \mathrm{SD}_{\mathrm{base}}$
k	$\mathbf{P}$[Obs. 1]	$\mathbf{P}$ if $k = 1$[Cor. 1], $\mathbf{NP}$-hard if $k = 2$[Th. 2]	$\mathbf{NP}$-hard if $k = 1$[Th. 3]
δ		$\mathbf{P}$ if $\delta = 0$[Th. 1]	$\mathbf{P}$ if $\delta < n - 1$[Th. 1]
		$\mathbf{NP}$-hard if $\delta \in \{1, k-1\}$[Th. 4]	$\mathbf{NP}$-hard if $\delta = (n-1)(k-1)$[Th. 5a]
$\max \lvert U(a)\rvert$		$\mathbf{P}$ if $\forall\, a \in A:\ \lvert U(a)\rvert = 1$[Obs. 1]	
		$\mathbf{NP}$-hard if $\max_{a \in A} \lvert U(a)\rvert \geq 2$[Th. 4b,Th. 5b]	
$\max \lvert\lvert U(a)\rvert - \lvert U(a')\rvert\rvert$		$\mathbf{P}$ if $\forall a, a' \in A : \lvert U(a)\rvert = \lvert U(a')\rvert$[Obs. 1]	
		$\mathbf{NP}$-hard if $\max_{a,a' \in A} \lvert\lvert U(a)\rvert - \lvert U(a')\rvert\rvert = 1$[Th. 4b,Th. 5b]	
m		Solvable in $f(m) \cdot (n \cdot m)^{O(1)}$ time for some function f [Cor. 2, Cor. 4]	
n		Solvable in $f(n) \cdot (n \cdot m)^{O(1)}$ time for some function f [Cor. 3, Cor. 4]	

agent satisfaction (Rawlsian [8]), mean satisfaction, and a variety of inequality indices including the mean/maximum absolute difference, variance, Schutz index, and coefficient of variation [25]. Widely used indices from economics, such as the Gini index, Atkinson index, and Robin Hood index, have been applied in allocation settings [2, 27, 11, 31, 29], and adapted for collective decision problems. These indices offer complementary perspectives, from minimizing dissatisfaction gaps (egalitarianism), optimizing total welfare (utilitarianism), or explicitly balancing inequalities, as formalized through measures like the Lorenz curve and Pigou-Dalton principle [11].

3 Complexity Results

First, we observe that γ-EEQI is polynomial-time solvable when measuring the satisfaction with $\mathrm{sat}_{\mathrm{norm}}$, regardless of which of the two equitability measures is used. This is also the case for $\mathrm{sat}_{\mathrm{base}}$ if all agents approve equally many candidates:

Observation 1 ($\star$). *There is an $r^+ \leq m$ and committee sequence $\mathcal{X} \in \mathcal{R}(\mathcal{E}, r^+)$ which solves SD_s-EEQI and MD_s-EEQI for any δ if (i) $s = \mathrm{norm}$ or (ii) $s = \mathrm{base}$ and $\exists \sigma \in \mathbb{N}\, \forall a \in A : \lvert U(a)\rvert = \sigma$.*

Remark 1. For these types of elections, the question raises whether there are fewer repetitions than r^+ so that each agent has the same satisfaction. Interestingly, this problem is $\mathbf{NP}$-hard, even if $\forall a \in A : \lvert U(a)\rvert = 3$, yet becomes polynomial-time solvable if each agent approves exactly two candidates.

From now on we measure the satisfactions with $\mathrm{sat}_{\mathrm{base}}$ and not with $\mathrm{sat}_{\mathrm{norm}}$, about which we know already that it is polynomial-time solvable in general.

3.1 The Committee Size and the Equitability Gap

Even if the agents' approval set sizes differ, both $\mathrm{MD}_{\mathrm{base}}$-EEQI and $\mathrm{SD}_{\mathrm{base}}$-EEQI are polynomial-time solvable if $\delta = 0$:

Theorem 1. $\mathrm{MD}_{\mathrm{base}}$-EEQI *is polynomial-time solvable if* $\delta = 0$ *and* $\mathrm{SD}_{\mathrm{base}}$-EEQI *is polynomial-time solvable if* $\delta < n - 1$.

Proof. Firstly, we claim that $\mathrm{SD}_{\mathrm{base}}$-EEQI with $\delta < n - 1$ is equivalent to $\mathrm{SD}_{\mathrm{base}}$-EEQI with $\delta = 0$. Let $\mathcal{X} = (X_1, \ldots, X_r)$ be a sequence of k-sized committees. Assume that there are a_1, a_2 with $\mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a_1) \neq \mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a_2)$. Let $A' \subseteq A$ be all agents $a \in A$ with $\mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a) = \mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a_1)$ and let $A'' \coloneqq A \setminus A'$. It holds that $\mathrm{SD}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}) \geq \sum_{a' \in A', a'' \in A''} \lvert \mathrm{sat}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}, a') - \mathrm{sat}_{\mathrm{s}}(\mathcal{E}, \mathcal{X}, a'')\rvert \geq \lvert A'\rvert \cdot \lvert A''\rvert \geq n - 1$. Thus, the claim follows.

We will exploit the following: If a linear program (LP)—i.e. the problem of finding an $x \in \mathbb{R}^n$ that maximizes $c^T x$ subject to $A'x \leq b$ and $x \geq 0$—has an optimal solution and each entry of A' is a rational number, then it also has an optimal solution x in which each entry of x is rational [19, Theorem 4.4].

For this, first note that, for both $\mathrm{SD}_{\mathrm{base}}$-EEQI and $\mathrm{MD}_{\mathrm{base}}$-EEQI, finding a committee sequence $\mathcal{X}$ so that $\delta = 0$ is equivalent to finding a committee sequence so that each agent has the same satisfaction, i.e. $\forall a, a' \in A : \mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a) = \mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a')$. We can formulate the problem as an integer linear program (i.e. an LP in which each entry of x is an integer) in which the variable x_c is the number of times the candidate c is chosen, the variable r the number of repetitions, and a is an arbitrary, but fixed agent:

$$\max 0 \text{ s.t.}$$

$$\forall a, a' \in A : \sum_{c \in U(a)} x_c - \sum_{c' \in U(a')} x_{c'} = 0 \tag{1}$$

$$\sum_{c \in C} x_c = k \cdot r \tag{2}$$

$$r \in \mathbb{N},\ \forall c \in C : x_c \in \{0, 1, \ldots, r\} \tag{3}$$

Constraints (1) ensure that each agent a' has the same satisfaction as agent a (as $\mathrm{sat}_{\mathrm{base}}(\mathcal{E}, \mathcal{X}, a) = \sum_{c \in U(a)} x_c$) and hence that each agent has the same satisfaction. Constraint (2) ensures that r times committees of size exactly k are elected. Constraints (3) ensure that there is at least one round and each candidate is elected at most once in each of the r repetitions. The objective is simply 0 as the problem at hand only asks whether there is a number of repetitions so that $\delta = 0$ and thus each feasible solution is also optimal.

We discuss the correctness of the ILP formulation next. If there is a committee sequence $(X_1, \ldots, X_r)$ witnessing that we have a `yes`-instance, then assigning $x_c = \lvert\{i \in \{1, \ldots, r\} \mid c \in X_i\}\rvert$ yields a solution to the ILP (one can directly verify that each of (1), (2), and (3) hold). For the other way around, let x_c, $c \in C$, be a solution to the ILP. We claim that there is a valid, equitable committee sequence $\mathcal{X} = (X_1, \ldots, X_r)$ such that $x_c = \sum_{i=1}^{r} X_{i,c}$ for every $c \in C$, where $X_{i,c}$ denotes the number of occurrences of c in X_i. Towards a contradiction suppose not, that is, for every such sequence $(X_1, \ldots, X_r)$ of k-sized multisets there is a $c \in C$ with $x_c > 0$ and an $i \in \{1, \ldots, r\}$ such that c appears at least twice in X_i. Let $\mathcal{X} = (X_1, \ldots, X_r)$ be a sequence that minimizes the number $\lvert M\rvert$ of multiple occurrences with $M \coloneqq \{(c, i) \mid$

c appears more than once in $X_i\}$. Let $(c,i) \in M$. Since $x_c \leq r$, there is a $j \in \{1,\dots,r\}$ such that $c \notin X_j$. Since $|X_i| = |X_j| = k$, there is a $c' \in X_j$ that either (i) is not in X_i, or (ii) is also in X_i and appears multiple times in X_j. We switch c from X_i with c' from X_j. Let M' denote M after the switch. In case (i), we have that $|M'| < |M|$. In case (ii), we have that $M' = M \setminus \{(c,i),(c',j)\} \cup \{(c',i)\}$ and hence $|M'| < |M|$. Either way contradicts the choice of $\mathcal{X}$, and thus the correctness follows.

Next, we look at the LP relaxation, which can be solved in polynomial-time, i.e. we allow the solution (i.e. the x_c and r) to be real numbers. If the LP is infeasible, so is the ILP. If the LP is feasible, we know that there is also a feasible, rational solution, because—when bringing the LP into standard form $A'x \leq b$ (with x consisting of the x_c and r)—all entries of A' are rational numbers (see above). Therefore, assume that we have a feasible solution to the LP described by rational numbers x_c and r and let, for a rational variable i, $i^{(n)} \in \mathbb{N}_0$ describe its numerator and $i^{(d)} \in \mathbb{N}$ its denominator, i.e. $i = \frac{i^{(n)}}{i^{(d)}}$. Let $l = \operatorname{lcm}\left(\bigcup_{c \in C} \left\{x_c^{(d)}\right\} \cup \left\{r^{(d)}\right\}\right)$ (with lcm being the least common multiple) and consider the solution $r' = l \cdot r \in \mathbb{N}_0$ and $x'_c = l \cdot x_c \in \mathbb{N}_0$ for all $c \in C$. It is easy to see that this is a feasible solution as well. □

Remark 2. The ILP described above can easily be changed for any δ for $\mathrm{MD}_{\mathrm{base}}$ and for $\mathrm{SD}_{\mathrm{base}}$. For $\mathrm{MD}_{\mathrm{base}}$, we replace (1) with

$$\forall a, a' \in A : \sum_{c \in U(a)} x_c - \sum_{c' \in U(a')} x_{c'} \leq \delta. \tag{4}$$

For $\mathrm{SD}_{\mathrm{base}}$, we introduce an integer variable $d_{\{a',a''\}}$ for each $\{a',a''\} \subseteq A$, which represents the absolute difference of the satisfaction of these two agents, and replace (1) with

$$\forall \{a,a'\} \subseteq A : \sum_{c \in U(a)} x_c - \sum_{c' \in U(a')} x_{c'} \leq d_{\{a,a'\}} \tag{5}$$

$$\sum_{\{a,a'\} \subseteq A} d_{\{a,a'\}} \leq \delta. \tag{6}$$

These are also the formulations we use in our experiments (see Section 4) by additionally minimizing δ or the number of repetitions r.

Next, we look at elections with $k = 1$: For $\mathrm{MD}_{\mathrm{base}}$-EEQI, planning only one repetition will lead to $\mathrm{MD}_{\mathrm{base}}(\mathcal{E},\mathcal{X}) \leq 1$ for any committee sequence $\mathcal{X} \in \mathcal{R}(\mathcal{E},1)$, as picking any candidate will lead to the agents approving this candidate having a satisfaction of one while the others have a satisfaction of zero. Therefore, each committee sequence in $\mathcal{R}(\mathcal{E},1)$ is a solution to our problem if $\delta \geq 1$. As we can determine in polynomial time whether there is a committee sequence so that everyone has the same satisfaction, i.e. for $\delta = 0$ (see Theorem 1), it follows:

Corollary 1. $\mathrm{MD}_{\mathrm{base}}$-EEQI *is polynomial-time solvable if* $k = 1$.

However, $\mathrm{MD}_{\mathrm{base}}$-EEQI becomes **NP**-hard if $k = 2$:

Theorem 2 (⋆). $\mathrm{MD}_{\mathrm{base}}$-EEQI *is* **NP***-hard if* $k = 2$.

This is in contrast to $\mathrm{SD}_{\mathrm{base}}$-EEQI, which is **NP**-hard when restricting the problem to elections with $k = 1$:

Theorem 3 (⋆). $\mathrm{SD}_{\mathrm{base}}$-EEQI *is* **NP***-hard even if* $k = 1$.

3.2 The Equitability Gap and the Maximum Size and Difference of the Approval Sets

Next, we show that $\mathrm{MD}_{\mathrm{base}}$-EEQI as well as $\mathrm{SD}_{\mathrm{base}}$-EEQI are **NP**-hard even if a) setting the upper bound δ to a relatively large value and if b) each agent approves one or two candidates (so that the difference in the number of approved candidates is at most one and two agents agree on at most one approved candidate):

Theorem 4 (⋆). $\mathrm{MD}_{\mathrm{base}}$-EEQI *is* **NP***-hard even if (a)* $\delta = k - 1$*; (b)* $\forall a \in A : 1 \leq |U(a)| \leq 2$ *and* $\delta = 1$.

Theorem 5 (⋆). $\mathrm{SD}_{\mathrm{base}}$-EEQI *is* **NP***-hard even if (a)* $\delta = (n-1)\,(k-1)$*; (b)* $\forall a \in A : 1 \leq |U(a)| \leq 2$.

3.3 The Number of Candidates and Agents

We distinguish between $\mathrm{MD}_{\mathrm{base}}$ and $\mathrm{SD}_{\mathrm{base}}$ next.

$\mathrm{MD}_{\mathrm{base}}$. The ILP given in Theorem 1, together with Remark 2 and Lenstra Jr. [24], yields the following.

Corollary 2. $\mathrm{MD}_{\mathrm{base}}$-EEQI *is fixed-parameter tractable when parameterized by the number* m *of candidates.*

The following reduction rule is immediate.

Reduction Rule 1 ($\mathrm{MD}_{\mathrm{base}}$-EEQI)**.** *If there are two agents approving exactly the same candidates, then delete one of them.*

Observation 2. *Let* I *be an instance of* $\mathrm{MD}_{\mathrm{base}}$-EEQI *where Reduction Rule 1 is inapplicable. Then there are at most* $2^m - 1$ *agents.*

We can modify the ILP to consider what we call *fingerprints*. Herein, $f\colon A \to \{0,1\}$ is the fingerprint of candidate c if $f(a) = 1 \iff c \in U(a)$. For fingerprint f, let m_f denote the number of candidates with fingerprint f, and let F denote the set of all occurring fingerprints. We then get the following ILP which is equivalent to the ILP composed of (2), (3), and (4).

$$\forall a, a' \in A : \sum_{f : f(a) = 1} x_f - \sum_{f : f(a') = 1} x_f \leq \delta$$

$$\sum_{f \in F} x_f = k \cdot r \tag{7}$$

$$r \in \mathbb{N},\ \forall f \in F : x_f \in \{0, 1, \dots, r \cdot m_f\} \tag{8}$$

Since there are at most 2^n pairwise different fingerprints, we get the following.

Corollary 3. $\mathrm{MD}_{\mathrm{base}}$-EEQI *is fixed-parameter tractable when parameterized by the number of pairwise different fingerprints and when parameterized by the number* n *of agents.*

$\mathrm{SD}_{\mathrm{base}}$. We introduce the following weighted variant of SD_{s}, where every agent a is equipped with a weight $w_a \in \mathbb{N}$: w-$\mathrm{SD}_{\mathrm{s}}(\mathcal{E},\mathcal{X}) = \sum_{\{a,a'\} \subseteq A} w_a \cdot w_{a'} \cdot |\mathrm{sat}_{\mathrm{s}}(\mathcal{E},\mathcal{X},a) - \mathrm{sat}_{\mathrm{s}}(\mathcal{E},\mathcal{X},a')|$. Note that $\mathrm{SD}_{\mathrm{base}}$-EEQI is the special case of w-$\mathrm{SD}_{\mathrm{base}}$-EEQI where $w_a = 1$ for every $a \in A$.

Reduction Rule 2 (w-$\mathrm{SD}_{\mathrm{base}}$-EEQI, ⋆)**.** *If there are two agents* a *and* a' *approving exactly the same candidates, then delete* a' *and let* $w_a \coloneqq w_a + w'_a$.

Observation 3. *Let* I *be an instance of* w-$\mathrm{SD}_{\mathrm{base}}$-EEQI *where Reduction Rule 2 is inapplicable. Then there are at most* $2^m - 1$ *agents.*

Together with the notion of fingerprints, the ILP for $\mathrm{SD}_{\mathrm{base}}$ (which is composed of (2), (3), (5), and (6)) can be hence adjusted as follows, where (7) and (8) are assumed to hold.

$$\forall\{a,a'\} \subseteq A: \sum_{f:f(a)=1} x_f - \sum_{f:f(a')=1} x_f \leq \frac{d_{\{a,a'\}}}{w_a \cdot w_{a'}}$$

$$\sum_{\{a,a'\}\subseteq A} d_{\{a,a'\}} \leq \delta$$

Corollary 4. $\mathrm{SD}_{\mathrm{base}}$-EEQI *is fixed-parameter tractable when parameterized by the number m of candidates and when parameterized by the number n of agents.*

4 Numerical Experiments about Equitability

In this section, we want to investigate experimentally what the best achievable δ is and how many repetitions are necessary for this δ.

4.1 Experimental Setup and Data

The experiments are based on three datasets consisting of approval elections: S_1 with $n = 30, m = 10, |S_1| = 569$, S_2 with $n = 50, m = 10, |S_2| = 770$, and S_3 with $n = 50, m = 20, |S_3| = 770$. We consider these relatively small dimensions, as we have shown that $\mathrm{MD}_{\mathrm{base}}$-EEQI is **NP**-hard even in a quite restricted setting and, in line with this, solving our implementation of these problems as ILPs in Python with the help of Gurobi is time-consuming.

As we are not aware of any publicly available real-world data with these dimensions, we generated these datasets with the help of the *Map of Elections* framework by using mainly the resampling model for approval elections with different parameter combinations [30]. This model can lead to agents approving nobody or candidates not approved by anybody: We remove such agents and candidates when searching for the most equitable committee sequence.

To visualize the elections in two-dimensional space, we plot each election as a point in a scatter plot. We define the coordinates of an election through an *explicit* function f mapping an election $\mathcal{E}$ to an x and y coordinate:

$$f(\mathcal{E}) \coloneqq (w(\mathcal{E}), e(\mathcal{E})) \text{ with } w(\mathcal{E}) \coloneqq \sum_{c\in C} n(\mathcal{E}, c) \text{ and}$$

$$e(\mathcal{E}) \coloneqq \max_{c'\in C} n(\mathcal{E}, c') - \min_{c''\in C} n(\mathcal{E}, c'').$$

Here, $n(\mathcal{E}, c) \coloneqq |\{a \in A : c \in U(a)\}|$ is the number of agents approving c, i.e. the approval score of candidate c. Therefore, $x = w(\mathcal{E})$ describes the sum of approval scores (i.e. the more agents approve more candidates, the larger x) and $y = e(\mathcal{E})$ is the maximum difference of the approval scores, i.e. equitability measure MD for approval scores instead of satisfactions. We will see that this function leads to the elections nicely covering the plane and that the results differ for different x and/or y values.

We will show results for S_3 and $k = 4$; analogous results for the other datasets and/or committee sizes in $[m-1]$ can be found in the full version of the paper, supporting the main observations.

4.2 Experimental Results

Most of the results for our EEQI problem when considering $\mathrm{sat}_{\mathrm{base}}$ and $k = 4$ are shown in Fig. 1. We want to highlight the following observations based on these plots (if we do not state whether an observation holds for $\mathrm{MD}_{\mathrm{base}}$ or $\mathrm{SD}_{\mathrm{base}}$, it holds for both):

- *What does the best (i.e. smallest) achievable δ, referred to as δ^*, look like for each election (see Fig. 1a, c)?* $\delta^* = 0$ is only achievable close to boundaries: When the sum of approval scores $x = w(\mathcal{E})$ is close to the highest value of $n \cdot m = 1000$, or when the maximal difference of the approval scores $y = e(\mathcal{E})$ is close to its highest value of $n = 50$ (i.e. when there is a candidate approved by (nearly) everyone and one approved by (nearly) no one), or when both x and y are close to the smallest value of 0 (i.e. when there are very few approvals overall).
  The elections for which δ^* is the worst lie in the lower half of the center—more precisely when $347 \leq w(\mathcal{E}) \leq 672$ and $9 \leq e(\mathcal{E}) \leq 28$—and δ^* tends to decrease when moving from this region towards different boundaries or corner points. With increasing k, the worst δ^* stays the same or increases, i.e. worsens, in our experiments.
- *How many repetitions r^* are necessary for δ^* (see Fig. 1b, d)?* For the majority of the elections, one repetition (i.e. $r^* = 1$) suffices. For $\mathrm{MD}_{\mathrm{base}}$ (Fig. 1b), there are 21 elections with $r^* > 1$: 17 of these elections are close to the origin, while four elections are closer to the center. These repetitions lead to an improvement of the equitability measure by one compared to only one repetition. This raises the question of whether this is not the consequence of $\mathrm{MD}_{\mathrm{base}}$ being quite coarse, since it only considers the maximum difference. Indeed, there are more than three times as many elections of this kind for $\mathrm{SD}_{\mathrm{base}}$ (Fig. 1d), more precisely 76. These elections are either relatively close to the origin or a bit closer to the center (with $w(\mathcal{E}) \leq 434$ and $e(\mathcal{E}) \leq 20$). These are regions in which some real-world elections end up, see Section 5.4.
  When increasing n or m in our experiments, the number of elections for which $r^* > 1$ often increases, but it decreases when increasing k: There are no elections with $r^* > 1$ for S_1—which is our smallest dataset with $n = 30, m = 10$—using $k = 5$ and $\mathrm{SD}_{\mathrm{base}}$, but there are always at least a few such elections for any k in our two other datasets with larger elections when using $\mathrm{SD}_{\mathrm{base}}$.

In addition, we looked at $\mathrm{sat}_{\mathrm{norm}}$, for which we know that we can ensure through $r^+ = \mathrm{lcm}(m^+, k)/k$ repetitions that every agent has the same satisfaction (see Observation 1). We investigated whether fewer repetitions than r^+ suffice to make every agent equally happy: For S_3 and $k = 4$, this is the case for 44 elections, which mainly have $e(\mathcal{E})$ values close to the maximal value.

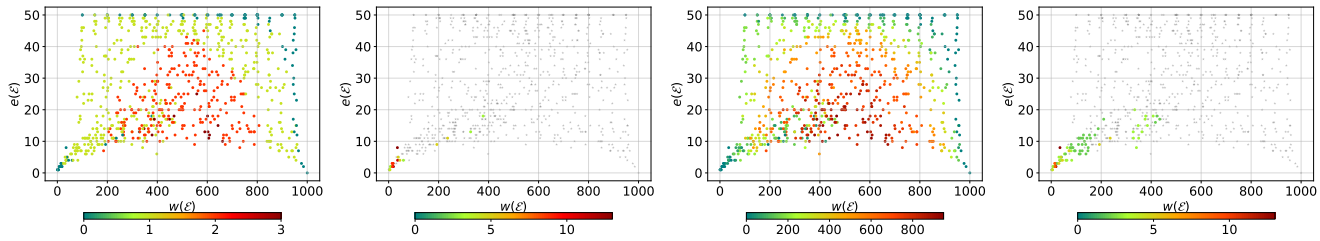
5 Explicit Map of Approval Elections

In this section, we want to compare our way of embedding approval elections into Euclidean space through the explicit function f (see Section 4.1) with the existing *Map of Approval Elections* [30], the idea of which is to embed elections in such a way that similar elections are close to each other. For this, Szufa et al. [30] defined a distance function that measures the distance between two elections and used multidimensional scaling to embed elections into a Euclidean plane so that the distance between each pair of elections is preserved approximately. We will therefore refer to this map as the *distance-embedding map* in contrast to our *explicit map* which maps an election into $\mathbb{R}$ directly through the function f.

5.1 General Advantages

First, we want to highlight the following general disadvantages of the distance-embedding approach:

- When some elections are changed, added, or removed, the same election can change its position as a consequence (see Fig. 2).



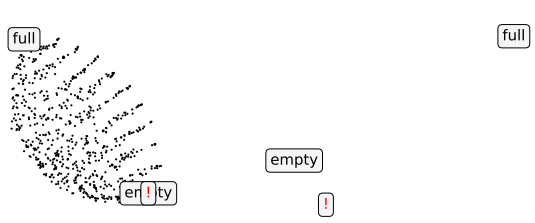
a) $\mathrm{MD}_{\mathrm{base}}$-EEQI: Best achievable δ.

b) $\mathrm{MD}_{\mathrm{base}}$-EEQI: Minimal r for best δ (gray stars mark $r = 1$).

c) $\mathrm{SD}_{\mathrm{base}}$-EEQI: Best achievable δ.

d) $\mathrm{SD}_{\mathrm{base}}$-EEQI: Minimal r for best δ (gray stars mark $r = 1$).

Figure 1. Each point represents one election from S_3 with $k = 4$, the position of which is determined via the explicit function f (see Section 4.1).



a) Elections from S_2.

b) Three elections from S_2.

Figure 2. Each point represents one election from the corresponding data set. On the left, all elections from S_2 (see Section 4.1) are visualized. On the right, the dataset consists of only three elections from S_2: The "empty" election, the "full" election and the election highlighted as "!". Both datasets are visualized using the distance-embedding map [30].

- To embed one election of interest on the map, further elections are needed because the embedding is based on the pairwise distances.
- The x and y coordinates in the map do not have a direct meaning and are therefore difficult to interpret.

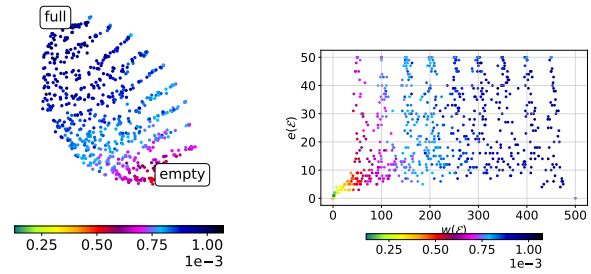
Our explicit map does *not* share these disadvantages, as the coordinates of an election only depend on the election itself and the coordinates have a direct meaning: the sum of approval scores and the maximal difference of two approval scores. A property—which may (not) be seen as a disadvantage depending on the application—of our explicit map is that two elections can have the same coordinate. If desired, one could use standard approaches or algorithms to fix or reduce such overplotting.

5.2 *Visualizing Features and Statistical Cultures*

Our explicit map is not only useful for visualizing the number of repetitions needed for a most equitable committee sequence (Section 4), but it also shares the following nice properties of the distance-embedding map presented in [30]: 1) The position of an election tells something about different quantitative properties (e.g. the runtime of the Proportional Approval Voting rule) and 2) elections generated with a statistical culture end up in certain regions on the map and the parameters of the statistical culture influence the election's position. These insights are helpful for picking the right statistical cultures and/or their parameters when planning experiments on elections with certain properties or when generating elections that should be similar to real-world instances of interest.

We visualize the quantitative properties considered in [30] for S_1, S_2, and S_3 in the full version of this paper. There, we also investigate the statistical cultures considered by Szufa et al. [30] for the datasets with $m = 50, n = 100$ that they made publicly available[2].

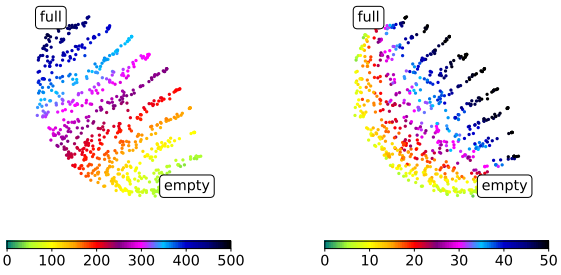
[2] The datasets used in [30] are published under https://github.com/Project-PRAGMA/Map-of-Approval-Elections--IJCAI-2022



a) Distance-embedding map

b) Explicit map

Figure 3. Each point represents one election from S_2 for $k = 5$. The color of each point represents the runtime of PAV.



The values of w.

The values of e.

Figure 4. Each point represents one election from S_2, which is visualized through the distance-embedding map from [30]. The color of each point represents the value of the corresponding measure.

Due to space limitations, nearly all the maps can only be found in the full paper, apart from the runtime of the Proportional Approval Voting rule (PAV) which returns the committees W with the highest $\sum_{a \in A} \sum_{j=1}^{|U(a) \cap W|} 1/j$ (implemented in `abcvoting` and solved using the Gurobi ILP solver) in Fig. 3. The reason for why our explicit map shares these nice properties with the distance-embedding map is that the values of w and e vary smoothly over the distance-embedding map and the directions of the color gradients are orthogonal when comparing w and e, as can be seen for S_2 in Fig. 4.

5.3 *Parameter Influence for the Resampling Model*

Furthermore, we show for each point (x, y) on our map that there are parameters p, ϕ of the resampling model introduced by Szufa et al. [30] so that, when generating elections with the (p, ϕ)-resampling model, the coordinates of the election are (x, y) in the limit. Szufa et al. [30] reason that in the limit, if $p \cdot m \in \mathbb{N}_0$, there are C_1 and C_2

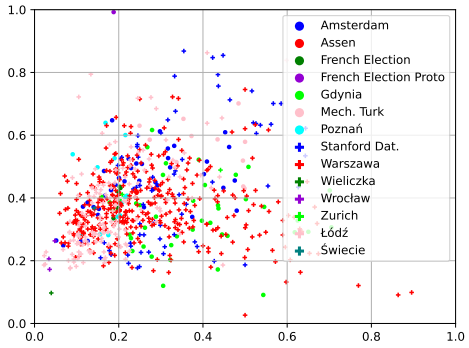


Figure 5. Each point on represents one real-world instance. Datasets are grouped based on their origin. The points are embedded using the general explicit map.

so that $C = C_1 \cup C_2$, $|C_1| = pm, |C_2| = (1-p)m, \forall c' \in C_1 : n(\mathcal{E}, c) = n\left((1-\phi) + (\phi p)\right), \forall c'' \in C_2 : n(\mathcal{E}, c) = n\phi p$. Based on this, it holds for our explicit map that

$$w(\mathcal{E}) = n\left(pm((1-\phi) + (\phi p)) + (1-p)\, m\phi p\right) = npm$$
$$e(\mathcal{E}) = n\left((1-\phi) + (\phi p) - (\phi p)\right) = n\left(1-\phi\right).$$

Therefore, if the goal is that the generated election has coordinates (x, y), p should be set to $\frac{x}{nm}$ and ϕ to $\frac{n-y}{n}$. Hence, the parameters p and ϕ make it possible to cover regions of interest and the whole explicit map can be covered with the resampling model.

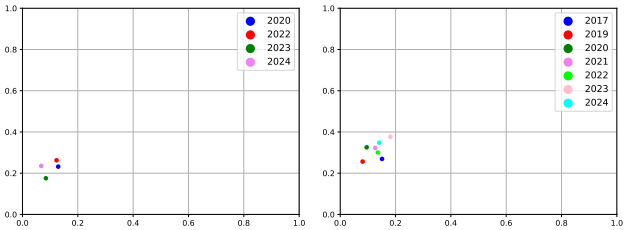
5.4 *Beyond Visualizing Instances with the Same Dimensions*

Visualizing instances with different dimensions is especially important when looking at real-world instances. However, the distance-embedding map is designed to visualize elections with the same dimension, and it is, to the best of our knowledge, unclear at the moment how to extend this distance-embedding approach to instances with different dimensions in a reasonable way. Because of that, Szufa et al. [30] visualized real-world instances from Pabulib [12] (a collection of participatory budgeting data) by first picking a fixed number of agents n and candidates m and then randomly selecting a subset of m candidates and n voters for each Pabulib election with at least n agents and m candidates. This led to the elections forming a contiguous region close to the empty election (i.e. close to the origin in our explicit map). This could give rise to the assumption that realistic elections (especially those for participatory budgeting) lie in this region.

In contrast, our explicit map can be easily extended to instances with differing dimensions by dividing the x and y measures by their maximal values, leading to $x, y \in [0, 1]$. In case of $w(\mathcal{E})$ the maximum is $n \cdot m$, in case of $e(\mathcal{E})$ it is n. Thus, our *general explicit map* is described through the explicit function $f(\mathcal{E}) = (w'(\mathcal{E}), e'(\mathcal{E}))$ with

$$w'(\mathcal{E}) := \frac{w(\mathcal{E})}{n \cdot m} = \frac{\sum_{c \in C} n(\mathcal{E}, c)}{n \cdot m}$$
$$e'(\mathcal{E}) := \frac{e(\mathcal{E})}{n} = \frac{\max_{c' \in C} n(\mathcal{E}, c') - \min_{c'' \in C} n(\mathcal{E}, c'')}{n}.$$

This general explicit map allows us to visualize real-world instances with differing dimensions in Fig. 5, which consists of Pabulib elections with approval preferences and two datasets [22, 23] with approval preferences from PrefLib [26] based on the French presidential election in 2002. These results show that the conclusion that real-world instances lie close to the origin is premature, as the real-world



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Figure 6. Each point on the general explicit maps represents one election from Pabulib collected in the corresponding subunit. The color of each point represents the year in which the dataset has been collected.

instances investigated cover larger parts of the map (the majority with $x \leq 0.4$ and $0.2 \leq y \leq 0.6$). Therefore, when aiming for elections that are similar to real-world elections of interest, first visualizing these instances on the map seems reasonable.

The dataset collected in Pabulib also allows investigating how elections that where carried out in the same city or district, but in different years, change over time. We visualize this for two subunits in Fig. 6 and provide maps for other subunits and/or cities for which this information is provided in Pabulib the full paper. These maps show that, for the majority of the subunits, the elections from different years end up (very) close to each other on the map, a reason of which might be that there could be a significant amount of people that participate in most or all of these elections because they live permanently in the subunit. This observation can be valuable in the emerging research about temporal elections, e.g. when making assumptions about the preferences of agents over time or when generating instances to conduct numerical experiments.

6 Epilogue

When allowing repeated committee elections, we can efficiently check whether a solution exists that satisfies every agent equally. If this is not possible, it turns into an **NP**-hard task to achieve the smallest difference among any two agents' satisfactions. This even holds true for small approval sets that differ little (but not for same-sized approval sets—then, the task becomes tractable). Interestingly, we leave open whether the problems are contained in **NP**. Since we proved the problems to be fixed-parameter tractable regarding the parameters number m of candidates and number n of agents, in future work, we also seek to find the smallest possible sizes of problem kernels regarding the two parameters.

For most of our tractable cases, a polynomial-time computable solution may have a large number of repetitions. We proved that whether we can reduce the number of repetitions keeping the same equitability bound is an **NP**-hard task, even in quite restricted settings. In future work, we plan to deepen our understanding of this problem's computational complexity.

Formulating the problems as an ILP, we were able to conduct experiments for a large set of data. We observed that several repetitions may be required to achieve the best equitability. Additionally, we found interesting correlations among the sum of approval scores, the difference of the most and least approved candidate, and our problems specifics, such as the best possible equitability value δ. The derived visualization shows several advantages over the distance-based map of approval elections, being a state-of-the-art visualization in such context.

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